

3. Given

$$e^{\frac{xy}{z}} \sqrt{xy^2z} - \cos(5yz) + 372xyz = 2xyz.$$

Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial y}{\partial z}$  by implicit differentiation. [4 marks]

4. Given  $f(x, y) = \frac{\ln(x^4)}{y^2} + x^2y^5 - 100$ , find the rate of change of the function  $f$  at the point  $(1, -1)$  in the direction of  $(5, -3)$ . [4 marks]

5. Given  $g(x, y, z) = xy^2z^3 - 17xz^{-2} + 9y^{-5}$ .

(a) Find the direction of the fastest decrease in the function  $g$  at the point  $(-1, 2, 1)$ ? [3 marks]

(b) Find the rate of change of  $g$  in the direction of the fastest decrease in values. [3 marks]

6. Find all critical points for optimising  $f(x, y) = 3x^4 + y^2 - x^2 - 2y$ . Classify these critical points (whenever possible) using the Hessian matrix to find relative extrema and saddle points of the function  $f$ . [5 marks]

8. The first five terms of a sequence are  $\frac{1}{5}, -\frac{2}{3}, \frac{4}{7}, -8, \frac{128}{9}, \dots$

Find an explicit formula for the general  $n^{\text{th}}$  term of the sequence? [2 marks]

9. Find the limit of the sequence  $\{a_n\}_{n=1}^{\infty}$  whose  $n^{\text{th}}$  term is given by

(a)  $a_n = \frac{25n^4}{100n^4 + 99n^2 + 1000}$

(d)  $a_n = \frac{17\sqrt[3]{n}}{\ln n}$

(b)  $a_n = \left( \frac{5^n}{6^n + 1} \right)$

(e)  $a_n = \frac{n \sin(\frac{\pi n}{2})}{n + 17}$

(c)  $a_n = \frac{10 + (-1)^n}{5n}$

(f)  $a_n = \left( \frac{1}{3}n \right)^{-\frac{3}{n}}$

(g)  $a_n = \left( 1 - \frac{11}{n+3} \right)^{n-2}$

[2 marks each = 14 marks]

10. Consider a square symmetric  $n \times n$  matrix  $A$ . Is the following statement true or false?

If  $A$  is positive-definite, then  $-4A$  is positive-definite.

State reasons for your answer.

[2 marks]

