

# Stats 125 - Assignment 1, Semester 1 2016

## Due 3pm, Tuesday 22 March

This assignment contributes 5% of your final grade.

Maximum possible marks 60.

**You must show all your working to get full credit on each problem. You may use any formulae you have been given in class.**

**While diagrams and tables are often helpful to understand a problem, to gain full marks in questions 2 to 8 you must (also) include appropriate and correct mathematical working and/or formulae.**

**Remember that you may discuss how to approach a question with others but all written work must be your own.**

**Refer to the Assignment Instructions on Canvas for information about setting out etc.**

1. Suppose my sample space consists of all current STATS125 students.

Let:

$K$  be the event that a randomly selected current STATS125 student has ever used Facebook,

$M$  be the event that a randomly selected current STATS125 student has ever used Instagram,  
and

$T$  be the event that a randomly selected current STATS125 student has ever used Snapchat.

- (a) Draw 4 Venn diagrams and use shading to identify the following sets:

- i.  $(K \cup M) \cap (K^c \cup M)$

- ii.  $(M \cup T) \cap (M^c \cup T) \cap (M^c \cup T^c)$

- iii. the event that a randomly selected current STATS125 student has used both Facebook and Instagram but has never used Snapchat.

- iv. the event that a randomly selected current STATS125 student has never used Facebook, Instagram or Snapchat.

- (b) Write (i) and (ii) more simply in symbols. Then for each describe the simplified event in words.

- (c) Write (iii) and (iv) in symbols.

[14 marks]

2. At a certain university it is estimated that 25% of the students live in the halls of residence, 12% of the students are from overseas and 10% of the students are from overseas and live in the halls of residence. Use this information to estimate the following probabilities.

- (a) What is the probability that a randomly chosen student lives in a hall of residence or is from overseas?

- (b) What is the probability that a randomly chosen student is neither from overseas nor lives in a hall of residence?

[4 marks]

3. Assume that you are a member of a small class of 50 students (30 males and 20 females). Suppose I wish to select 10 students to take part in a survey.
- (a) In how many different ways can I select my 10 students?
  - (b) In how many different ways can I select 5 males and 5 females?
  - (c) What is the probability that I select 5 males and 5 females?
  - (d) Suppose I select the 10 students using a simple random sample. What is the probability that you are one of the 10 students?
  - (e) Are you a male or a female?
  - (f) Suppose instead I randomly select 5 of the males and 5 of the females. What is the probability that you are one of the 10 students?
- [10 marks]**

4. Suppose that  $\mathbb{P}(A) = 0.45$  and  $\mathbb{P}(B) = 0.26$ .
- (a) If  $A$  and  $B$  are mutually exclusive find  $\mathbb{P}(A \cap B)$  and  $\mathbb{P}(A \cup B)$ .
  - (b) If  $B \subset A$  find  $\mathbb{P}(A \cap B)$  and  $\mathbb{P}(A \cup B)$ .
  - (c) Is  $A \subset B$  possible? Explain why or why not.
  - (d) Can  $\mathbb{P}(A \cup B)$  ever be 1? If yes state when, if not give the largest value  $\mathbb{P}(A \cup B)$  can take and when it occurs.
  - (e) Can  $\mathbb{P}(A \cap B)$  ever be 0? If yes state when, if not give the smallest value  $\mathbb{P}(A \cup B)$  can take and when it occurs.
  - (f) If  $A$  and  $B$  are independent find  $\mathbb{P}(A \cap B)$  and  $\mathbb{P}(A \cup B)$ .
  - (g) Suppose also that  $\mathbb{P}(A \cap B) = 0.15$ . Find:
    - i.  $\mathbb{P}(A|B)$
    - ii.  $\mathbb{P}(B^c|A)$
- [16 marks]**

5. If  $\mathbb{P}(A|B) > \mathbb{P}(A)$  show that  $\mathbb{P}(B|A) > \mathbb{P}(B)$ . (You may assume that both  $\mathbb{P}(A) > 0$  and  $\mathbb{P}(B) > 0$ .)
- [3 marks]**

6. Haemophilia is a genetic disease that affects the blood's clotting ability. It is caused by a recessive gene on the X chromosome. For the purposes of this question you may assume that haemophilia can only occur through heredity and not through mutation. A woman is said to be a carrier of haemophilia if she has the haemophilia gene on one of her two X chromosomes. A woman carrier has a probability of 0.5 of transmitting the disease to each son, since a son will get an X chromosome from his mother and a Y chromosome from his father. A male has haemophilia if their X chromosome has the haemophilia disease, thus they can only have haemophilia if their mother is a carrier or alternatively has haemophilia themselves.

Because of her family history, prior to having any children a woman is thought to have a 40% chance of being carrier. (Note: you may assume that the woman does not have haemophilia and thus the only possibilities are that she is a carrier or not a carrier.)

- (a) Suppose that this woman gives birth to a son. What is the probability that he will have haemophilia?
- (b) Suppose the woman has a son who does not have haemophilia.
  - i. Based on this new information, what is the probability that she is a carrier?
  - ii. Suppose that she now gives birth to another son what is the probability that he too will not have haemophilia?
- (c) Now suppose that the woman has two sons, neither of whom have haemophilia. Based on this information, what is the probability that she is a carrier?

[10 marks]

7. Suppose that  $\mathbb{P}(A) = 0.1$ ,  $\mathbb{P}(B) = 0.2$  and  $\mathbb{P}(C) = 0.5$  and that  $A$ ,  $B$  and  $C$  are mutually independent. find:

- (a)  $\mathbb{P}(A \cap B \cap C)$
- (b)  $\mathbb{P}(A \cup B \cup C)$

[3 marks]

8. *This is a bonus question. You will get extra marks for doing it.*

- (a) Consider a class of 30 students. Which is more likely, at least two students share the same birthday or at least one student was born on January 1st? Justify your answer. (Your justification should include the calculation of both probabilities and an explanation as to why.)
- (b) Consider a class of  $n$  students. Which is more likely, at least two students share the same birthday or at least one student was born on January 1st? Does it make any difference what  $n$  is? Justify your answer.

[4 bonus marks]