Problem Set 4

MSU EC 410
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1. Imagine there are two types of potential borrowers in a village, those with p_i=0.7 and those with p_i=0.9. As in the model discussed in class, these borrowers succeed with probability p_i and fail with probability $(1-p_i)$. If they fail, they get no payoff and can pay nothing to the bank (*there is no collateral*). If they succeed, they pay back (1+r)L to the bank.
Assume L = \$100. Assume the expected *gross* payoff (i.e. without accounting for payment to the bank) to borrowing is \$200 for *all* borrowers.
Assume all potential borrowers have a non-borrowing option they can choose instead of borrowing: working for a subsistence wage of \$70.
First consider *individual* (not *group*) lending.

a. Write down the expected payment made to the bank by a borrower with p_i=0.9. Write down the expected payment made to the bank by a borrower with p_i=0.7. Leave both answers in terms of r.

b. Assume the bank can observe the individuals' risk-types, because of credit history for example, and can charge different borrowers different interest rates. If the bank must earn an expected return of 10% on its loans, what interest rate would it charge a borrower with p_i=0.9? with p_i=0.7? Justify your answer.

c. For both borrowers, what is their expected *net* payoff to borrowing if the bank charges the interest rates of part b.? Which types will borrow (comparing the payoff to borrowing to the outside payoff)? Justify your answer.

d. Now assume the bank cannot observe the individuals' risk-types and must charge everyone the same interest rate. Assume the interest rate is the one charged to the risky type (with p_i = 0.7) in part b. (If you did not get part b., assume an interest rate of r = 57%.) Write down the expected payment made to the bank by a borrower with p_i=0.9. Write down the expected payment made to the bank by a borrower with p_i=0.7.

e. For both borrowers, what is their expected *net* payoff to borrowing? Which types will borrow (comparing the payoff to borrowing to the outside payoff)? Justify your answer.

f. In words, what is the adverse selection problem, when does it appear, and why?

Now consider group lending. Borrowers form pairs homogeneously. If a borrower succeeds, he pays (1+r)L. In addition, if he succeeds and his partner fails, he makes an extra payment of cL. Assume projects succeed or fail independently of each other. Now assume r = 30% and c = 90%, that is, 90% of the partner's loan principal must be paid by a borrower who succeeds and whose partner fails.

g. Find the effective interest rate for a borrower with p_i=0.9. Find the effective interest rate for a borrower with p_i=0.7. For each, how does this interest rate compare with that of parts b. and d.?

h. For both borrowers, what is their expected *net* payoff to borrowing? [Hint: expected repayment to the bank is just $p_i(1+\tilde{r}_i)L$, where $\tilde{r}_i$ is the effective interest rate for borrower i.] Which types will borrow (comparing the payoff to borrowing to the outside payoff)? Justify your answer.

i. In words, briefly explain how group lending can overcome the adverse selection problem.

2. Project A pays \$75,000 (i.e. succeeds) with probability 9/10 and pays 0 (i.e. fails) with probability 1/10. Project B pays \$100,000 (i.e. succeeds) with probability 3/5 and pays 0 (i.e. fails) with probability 2/5.
[Note: the *expected value* for any random amount X, that takes on values $\{x_1, x_2, ..., x_N\}$ with probabilities $\{p_1, p_2, ..., p_N\}$ respectively, equals $\sum_{i=1}^{N} p_i\, x_i$. For instance, if profits equal \$40 with probability 1/3, \$60 with probability 1/2, and 0 with probability 1/6, then the expected value of profits (i.e. expected profits) equals
1/3 * \$40 + 1/2 * \$60 + 1/6 * 0 = \$43.33.]

a. What is the expected gross payoff, i.e. expected gross value, of project A? What is the expected gross payoff, i.e. expected gross value, of project B?

b. Assume each project requires an investment of $55,000 to undertake. Consider a person with $55,000 to invest who just cares about maximizing *expected* payoffs. Would this person prefer to invest in project A or B?

Now consider a person who is borrowing the $55,000 in order to invest in project A or B. Assume the interest rate charged is r = 8%, whichever project is chosen. Assume the borrower puts up collateral C and faces no liability other than that if the project fails. Thus, if the project fails, the borrower loses C (this is equivalent to paying C); if the project succeeds, the borrower repays (1+r) $55,000.

c. What is the borrower's expected payment to the bank (counting collateral seized as making a payment) under project A? under project B? Leave your answers in terms of C.

d. What is the borrower's expected net payoff under project A? under project B? Leave your answers in terms of C.

e. If C = 0 and the borrower maximizes expected payoffs, which project will the borrower choose?

f. If C = (1+r)$55,000 and the borrower maximizes expected payoffs, which project will the borrower choose?

g. What is the minimum collateral level necessary for the borrower to make the same choice as if his own money were at stake (i.e. the answer of part b.)?

h. In words, what is causing the moral hazard problem here?

Consider now joint liability. There is now a group of two, each of which is liable for the other's loan. Specifically, if one succeeds and the other fails, the one who succeeds owes an extra $16,500 (i.e. 30% of the loan principal, c=30%). The interest rate r is now reduced to compensate for the extra liability: r = 5%. [This keeps the bank's expected profits the same as before provided borrowers choose the safe project, as you could check.] To review, now if the borrower succeeds and his partner succeeds, he repays $57,750 (= [1+5%] * $55,000); if he succeeds and his partner fails, he repays $74,250; and if he fails, he loses collateral C.

i. What is a borrower's expected payment to the bank (counting collateral seized as making a payment) if both he and his partner choose project A? if they both choose project B? Leave your answers in terms of C. [Hint: Remember that the expected repayment is the amount paid in each state of the world multiplied by the probability of being in that state of the world. For both succeeding, the probability is just p^2; for one succeeding and the other failing, it is $p*(1–p)$; and for failing, it is $(1–p)$.]

j. What is the borrower's expected net payoff if both group members choose project A? if both group members choose project B? Leave your answers in terms of C.

k. What is the minimum collateral level necessary for a pair of borrowers that choose the same project to make the same choice as if their own money were at stake (i.e. project A)?

l. In words, how does joint liability affect the moral hazard problem here?

3. Here we explore how dynamic lending can reduce limited enforcement. In particular, consider the model introduced in class, where the borrower repays when and only when the financial benefit of the future relationship, $B(r_{t+1},L_{t+1},\ldots,r_T,L_T)$, is at least as great as the financial cost of repaying the current loan, $(1+r_t)L_t$. Let t=0 and T=3 – that is, the borrower anticipates 3 more loans after repaying the current one. Assume that the borrower has a project that always doubles the value of the capital input, without risk – e.g. a $100 loan can be turned into a certain $200 gross return. If the borrower repays, the net return would then subtract off the cost of the capital, principal and interest – e.g. if r=20% and L=$100, the cost of capital would be $120 and the net return would be $200-$120 = $80. Thus the value of one loan would be $80. In calculating benefits of future loans, assume the borrower does not discount the future (i.e. values current and future income equally) and that he plans to repay all future loans when deciding whether to repay the current loan.

a. Assume all loans sizes and interest rates are the same: $L_0=L_1=L_2=L_3=\$100$ and $r_0=r_1=r_2=r_3=r$, say. At what interest rates r would the borrower choose to repay the current (0^{th}) loan?

b. Assume $r_0=r_1=r_2=r_3=60\%$, and compare repayment behavior when $L_0=L_1=L_2=L_3=\$100$ to when $L_0=\$25$, $L_1=\$75$, $L_2=\$125$, $L_3=\$175$. In each case, which loan will the borrower default on?

c. Now add the expectation of a 4th loan: i.e. T=4. Consider $r_0=r_1=r_2=r_3=r_4=60\%$ and $L_0=L_1=L_2=L_3=L_4=\$100$. Will the borrower repay the current (0th) loan? How does this compare to repayment behavior under these loan sizes and interest rates when T=3?

d. Return to the assumptions of part a. However, assume that capital is worth less to borrowers, so that their projects only increase the value of the capital by 50% rather than doubling it. (E.g. a \$100 loan grosses \$150 with certainty, rather than \$200.) At what interest rates r would the borrower choose to repay the current (0th) loan?