Problem Set 3

MSU EC 410
Prof. Ahlin
due 11/10/15

1a. Use the H-augmented Solow model to determine the a) instantaneous impact on GDP per capita, b) instantaneous impact on consumption per capita, c) long-run impact on GDP per capita, d) long-run impact on consumption per capita, e) impact on long-run GDP per capita growth rate, and f) impact on long-run GDP growth rate of a *permanent and instantaneous increase in the fraction of national resources devoted to investment in human capital*, *q*. Assume the country begins at its steady state values of k^* and h^* before this event occurs. Justify your answer by use of graph and/or equation.

1b. How does each answer compare to the answer the original Solow model would give when s increases, both qualitatively (*whether* the amount goes up or down) and quantitatively (the *amount by which* it goes up or down)?

2. Consider the Solow model with total factor productivity A_t constantly growing at rate $g>0$.

a. Determine the a) instantaneous impact on GDP per capita, b) instantaneous impact on consumption per capita, c) long-run impact on GDP per capita (i.e. compare the level of GDP per capita with and without the parameter change, in the long-run), d) long-run impact on consumption per capita (i.e. compare the level of consumption per capita with and without the parameter change, in the long-run), and e) impact on long-run GDP per capita growth rate **of a *one-time and instantaneous increase (jump) in productivity A_t***, through a significant and non-repeatable invention. Assume the country begins at its "steady state value" of k^* before this event occurs. Justify your answer by use of graph and/or equation. [Hint: this should not be considered a change in g, since productivity resumes growth at rate g after the one-time jump; it should be modeled as a one-time jump in A_t.]

b. Graph the path of y_t and c_t against time (or better yet, $\ln(y_t)$ and $\ln(c_t)$, which will be linear) for the event analyzed in part a.

c. Repeat parts a&b for a ***permanent, instantaneous increase in the growth rate of productivity, g***.

3. Growth Simulations. See PS3GrowthSimulationQuestion.xlsx posted on D2L. Fill in 200 years of data using the H-D model, the Solow model, and the H-Solow model using the functions and parameters given in the "GrowthCalculations" worksheet. The savings rate in all cases increases to 30% at year 25.
Specifically:

3.1. For the H-D model, A=0.25, n=0.01, d=0.04, and s=0.2. Capital per person starts at \$4000. Fill out $\boldsymbol{k}$, $\boldsymbol{y}$, $\ln(\boldsymbol{y})$, $\boldsymbol{c}$, $\ln(\boldsymbol{c})$, actual investment, break-even investment, $\Delta\boldsymbol{k}$, and $g_{\boldsymbol{y}}$ for 200 years.

3.2. For the Solow model, $f(\boldsymbol{k}) = \boldsymbol{k}^{1/3}$, A=50, n=0.005, d=0.02, and s=0.2. Capital per person starts at \$8000. Fill out $\boldsymbol{k}$, $\boldsymbol{y}$, $\boldsymbol{c}$, actual investment, break-even investment, $\Delta\boldsymbol{k}$, and $g_{\boldsymbol{y}}$ for 200 years.

3.3. For the H-Solow model, $f(\boldsymbol{k},\boldsymbol{h}) = \boldsymbol{k}^{1/3}\boldsymbol{h}^{1/3}$, A=5, n=0.01, d=0.04, and s=0.2. Physical capital per person starts at \$4000, human capital per person starts at 2000. Fill out $\boldsymbol{k}$, $\boldsymbol{h}$, $\boldsymbol{y}$, $\boldsymbol{c}$, $\Delta\boldsymbol{k}$, $\Delta\boldsymbol{h}$, and $g_{\boldsymbol{y}}$ for 200 years.

Note that in all cases, the savings rate s switches at 0.3 at the 25th year. Make sure to incorporate this in your answers. [Hint: it will only affect the consumption formula and the actual investment column formula for the H-D and Solow models, and it will only affect the consumption formula and the $\Delta\boldsymbol{k}$ column formula for the H-Solow model.] [Hint: Of course, you need only specify each column's formula once, then copy and paste down the column for all the years. The formulas are pretty straightforward, and can be found by looking back at the key equations for each model. It is simplest for actual investment not to recalculate income, but simply use the fact that actual investment equals a fixed fraction of income, $s\boldsymbol{y}$ in the case of physical capital and $q\boldsymbol{y}$ in the case of human capital.]

a. Give the income and consumption levels in year 200 for each of the three models. In which model is the increase in s most effective? In which model is it least effective? Justify your answer.
b. Look at the graphs for the three models (which are in the other worksheets and should be filled out automatically from the data you generate in the GrowthCalculations worksheet). Look at both H-D graphs, but focus on the one using logs. Discuss one significant way in which all three models' graphs are similar. How do the Solow and H-Solow graphs differ?

4. Imagine that a bank will only lend if it can earn a rate of return of 6% on a loan. Further, imagine it incurs administrative costs of $40 per loan it makes, regardless of the size of the loan. Throughout the problem, assume for simplicity that the loans are all repaid with certainty, i.e. there is no risk.
a. If the bank makes five loans – of $100, $200, $500, $1000, and $10,000 – what are the respective interest rates it must charge to break even on each loan?
b. Imagine the bank makes the same loans but must charge all borrowers the same interest rate. What interest rate will it charge to break even overall? Which borrowers pay less, which pay more in this case than in part a.? This practice of making losses on some loans and profits on others is called "cross-subsidization".
c. How might competition between banks eliminate any one bank's ability to cross-subsidize smaller borrowers? Specifically, ci) could a rival lender lure away any of the customers of a bank carrying out the policy of part b., and cii) how would this affect the ability to cross-subsidize of a bank carrying out the policy of part b.?
d. It may not be accurate to assume that every loan incurs the same administrative cost, irrespective of size. Larger loans may require more work. Redo part a. under the assumption that the administrative cost of a loan is $40 per loan *plus* 1% of the size of the loan. (Thus a loan of $5000 would cost the bank $40 + 1%*$5000 = $90, while a loan of $500 would cost the bank $40 + 1%*$500 = $45. The cost structure is still linear, but with a positive intercept *and* slope.)