MATH 107 FINAL EXAMINATION

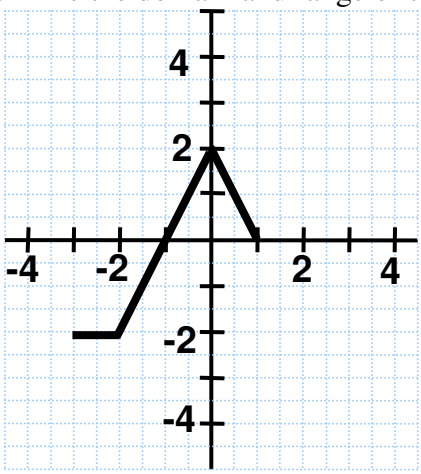
This is an open-book exam. You may refer to your text and other course materials as you work on the exam, and you may use a calculator. **You must complete the exam individually. Neither collaboration nor consultation with others is allowed.**

Record your answers and work on the separate answer sheet provided.

There are 30 problems.
Problems #1–12 are Multiple Choice.
Problems #13–21 are Short Answer. (Work not required to be shown)
Problems #22–30 are Short Answer with work required to be shown.

MULTIPLE CHOICE

1. Determine the domain and range of the piecewise function. 1. _____



A. Domain [–3, 1]; Range [–2, 2]
B. Domain [–2, 2]; Range [–3, 1]
C. Domain [–2, 0]; Range [0, 2]
D. Domain [–1, 1]; Range [0, 2]

2. Solve: $\sqrt{6x+27} = -x$ 2. _____

A. –3
B. –3, 9
C. –27/7
D. No solution

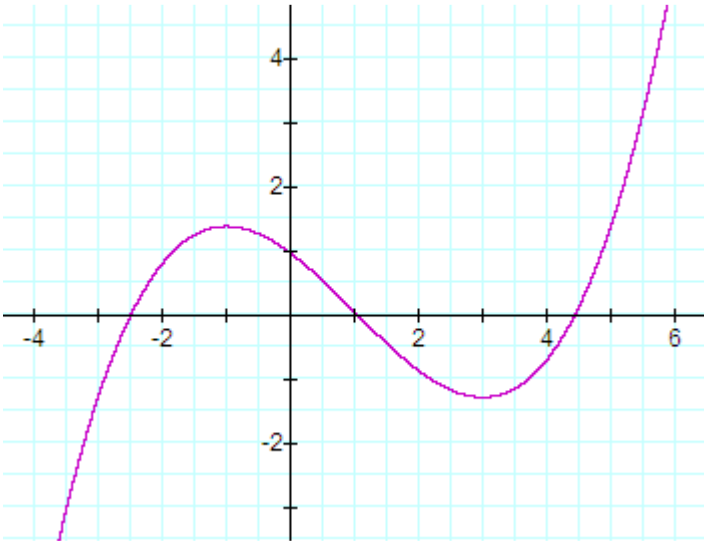
3. Determine the interval(s) on which the function is increasing. 3. _____

A. $(-1.3,\ 1.3)$

B. $(1,\ 3)$

C. $(-\infty, -1)$ and $(3, \infty)$

D. $(-2.5,\ 1)$ and $(4.5, \infty)$



4. Determine whether the graph of $y = |x| + 5$ is symmetric with respect to the origin, the *x*-axis, or the *y*-axis. 4. _____

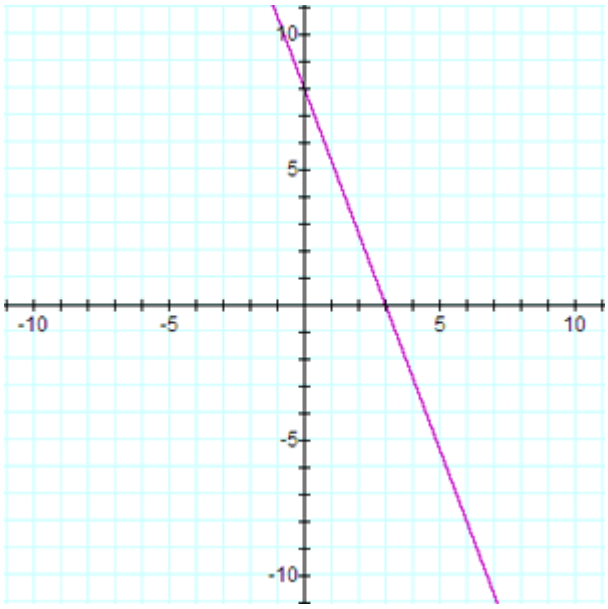
A. symmetric with respect to the *y*-axis only
B. symmetric with respect to the *x*-axis only
C. symmetric with respect to the origin only
D. not symmetric with respect to the *y*-axis, not symmetric with respect to the *x*-axis, and not symmetric with respect to the origin

5. Solve, and express the answer in interval notation: $| 6 - 5x | \leq 4$. 5. _____

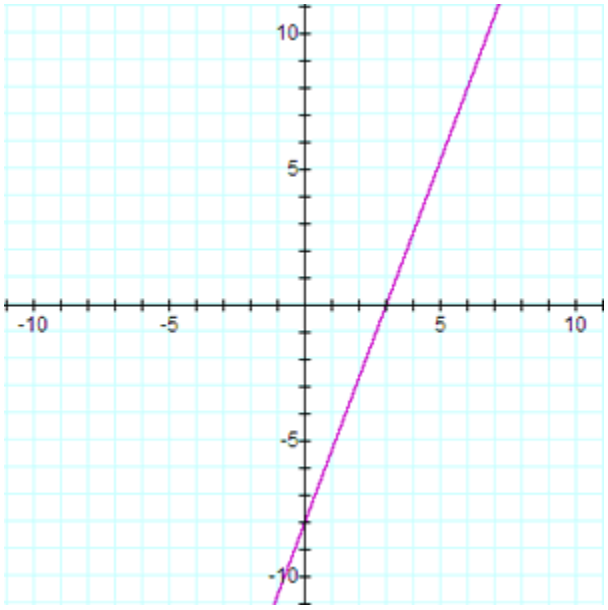
A. $(-\infty, 2/5]$
B. $[2, 2/5]$
C. $[2/5, 2]$
D. $(-\infty, 2/5] \cup [2, \infty)$

6. Which of the following represents the graph of $3x - 8y = 24$? 6. _____

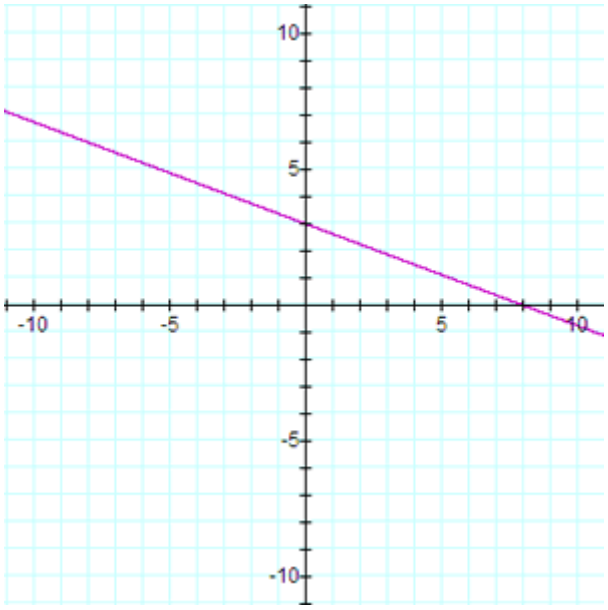
A.



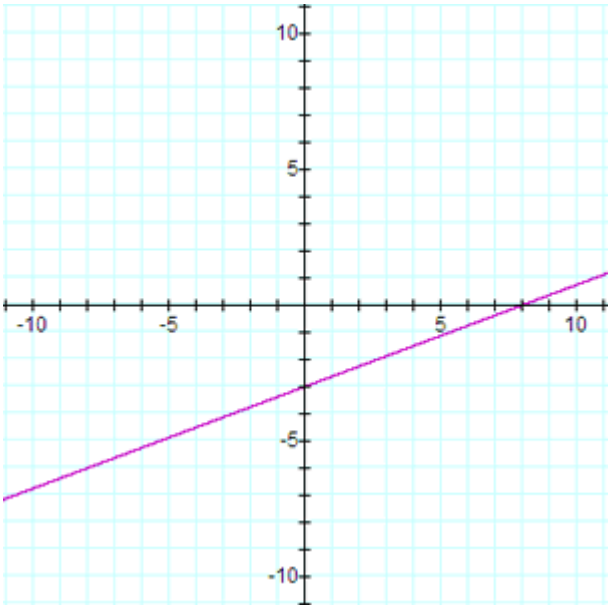
B.



C.



D.



7. Write a slope-intercept equation for a line parallel to the line $x + 3y = 8$ which passes through the point $(-4, 5)$. 7. _____

A. $y = \frac{1}{3}x + 5$

B. $y = -3x - 7$

C. $y = -\frac{1}{3}x + 5$

D. $y = -\frac{1}{3}x + \frac{11}{3}$

8. Which of the following best describes the graph? 8. _____

A. It is the graph of an absolute value relation.
B. It is the graph of a function and it is one-to-one.
C. It is the graph of a function but not one-to-one
D. It is not the graph of a function.

9. Express as a single logarithm: $4 \log x + \log 1 - \log y$ 9. _____

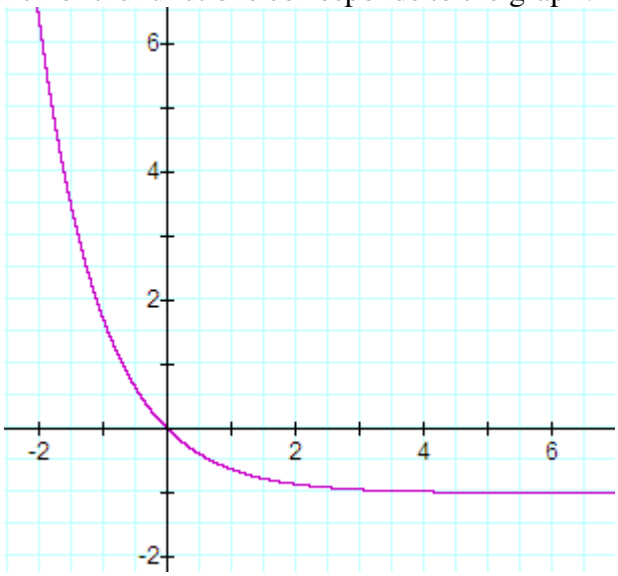
A. $\log\left(\frac{x+1}{y}\right)^4$

B. $\log\left(\frac{x^4}{y}\right)$

C. $\log(4x+1-y)$

D. $\log\left(\frac{4x+1}{y}\right)$

10. Which of the functions corresponds to the graph? 10. _____



A. $f(x) = -e^x$

B. $f(x) = e^{-x} - 1$

C. $f(x) = e^x - 2$

D. $f(x) = e^{-x} + 1$

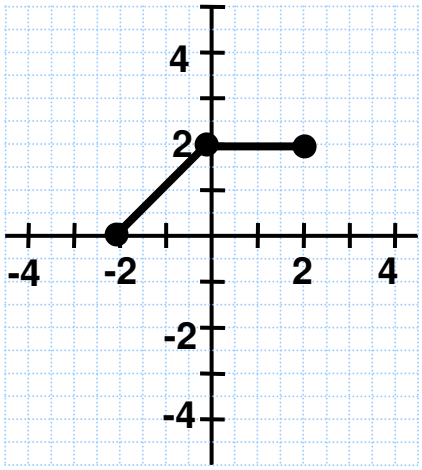
11. Suppose that for a function f, the equation $f(x) = 0$ has no real-number solutions.

Which of the following statements MUST be true? 11. _____

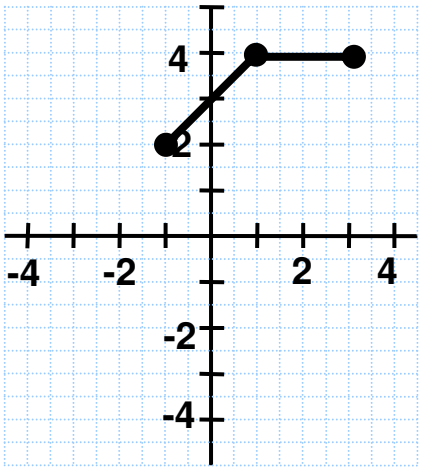
A. f has exactly no x-intercepts.
B. f has exactly no y-intercepts.
C. f is an invertible function.
D. The graph of f is a horizontal line.

12. The graph of $y = f(x)$ is shown at the left and the graph of $y = g(x)$ is shown at the right. (No formulas are given.) What is the relationship between $g(x)$ and $f(x)$?

12. _____



$y = f(x)$



$y = g(x)$

A. $g(x) = f(x - 2) + 1$
B. $g(x) = f(x - 1) + 2$
C. $g(x) = f(x + 1) + 2$
D. $g(x) = f(x + 2) + 1$

SHORT ANSWER:

13. Multiply and simplify: $(4 + 7i)(6 + 5i)$.
Write the answer in the form $a + bi$, where a and b are real numbers. Answer: _____

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14. Solve, and write the answer in interval notation: $\frac{x+2}{x-9} \le 0$. Answer: _____

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15. A bowl of soup at 200° F. is placed in a room of constant temperature of 60° F. The temperature T of the soup t minutes after it is placed in the room is given by

$$T(t) = 60 + 140\, e^{-0.075\, t}$$

Find the temperature of the soup 30 minutes after it is placed in the room. (Round to the nearest degree.)

Answer: _____

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16. Find the value of the logarithm: $\log_5\left(\frac{1}{125}\right)$. Answer: _____

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17. Solve: $8^{7x-2} = 64$. Answer: _____

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18. Suppose $8,800 is invested in an account at an annual interest rate of 6.2% compounded continuously. How long (to the nearest tenth of a year) will it take the investment to double in size? Answer: _____

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19. Let $f(x) = x^2 + 14x + 44$.

(a) Find the vertex. Answer: _____

(b) State the range of the function. Answer: _____

(c) On what interval is the function decreasing? Answer: _____

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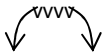
20. Consider the polynomial $P(x)$, shown in both standard form and factored form.

$$P(x)=\frac{1}{2}x^4+\frac{1}{2}x^3-\frac{7}{2}x^2-\frac{1}{2}x+3=\frac{1}{2}(x+3)(x+1)(x-1)(x-2)$$

(a) Which sketch illustrates the end behavior of the polynomial function?

A.	B.	C.	D.
			

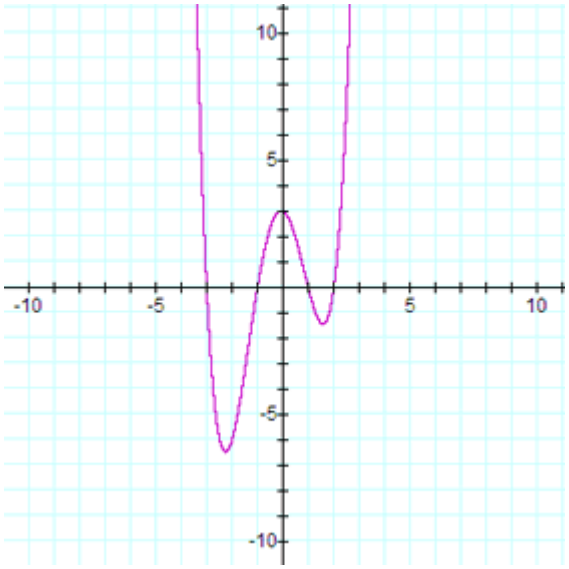
Answer: _____

(b) State the zeros of the function. Answer: _____

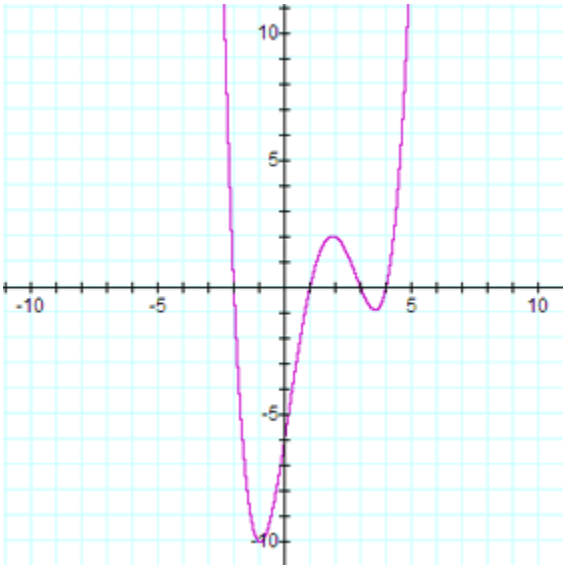
(c) State the y-intercept. Answer: _____

(d) State which graph below is the graph of $P(x)$. Answer: _____

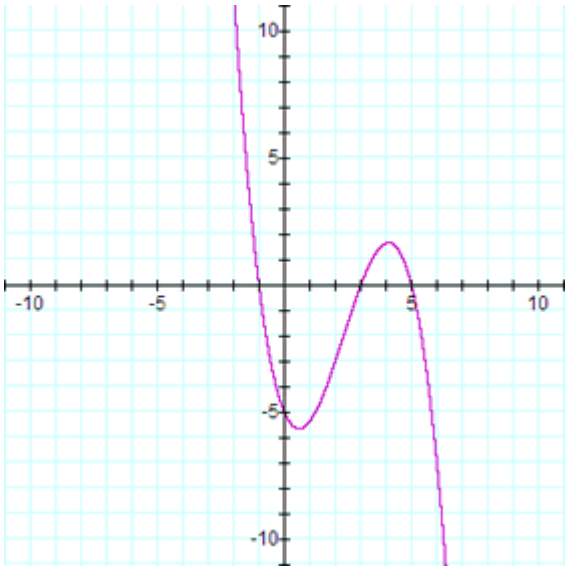
GRAPH A



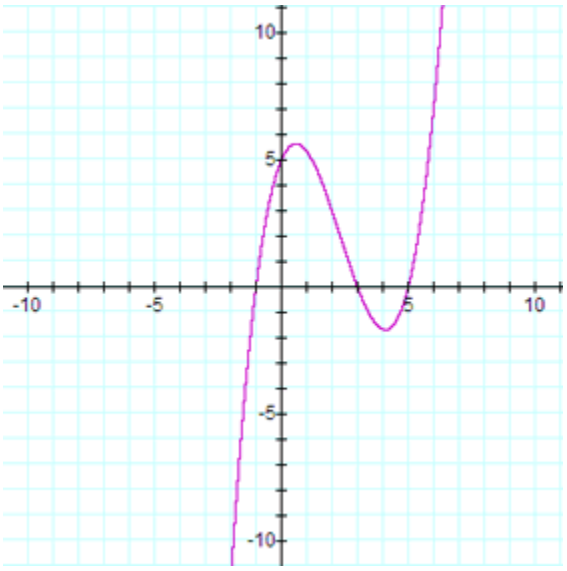
GRAPH B



GRAPH C



GRAPH D



21. Let $f(x)=\dfrac{2-x}{3+x}$.

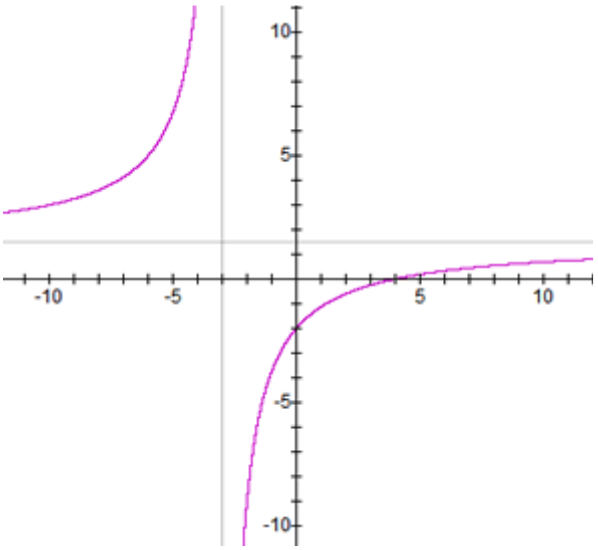
(a) State the domain. Answer: _____

(b) State the horizontal asymptote. Answer: _____

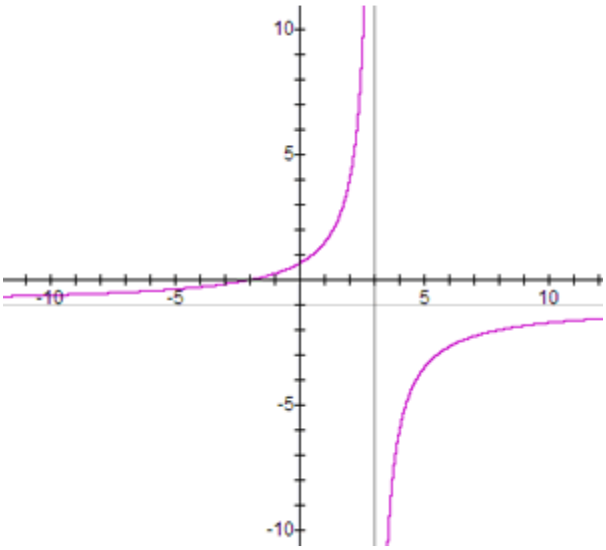
(c) State the vertical asymptote(s). Answer: _____

(d) Which of the following represents the graph of $f(x)=\dfrac{2-x}{3+x}$? Answer: _____

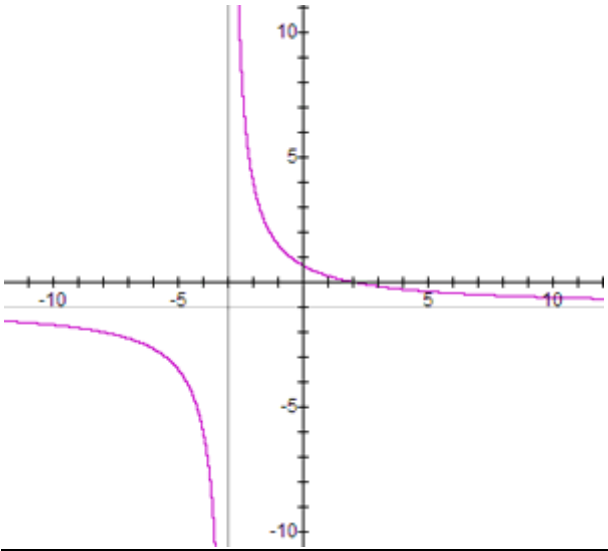
GRAPH A.



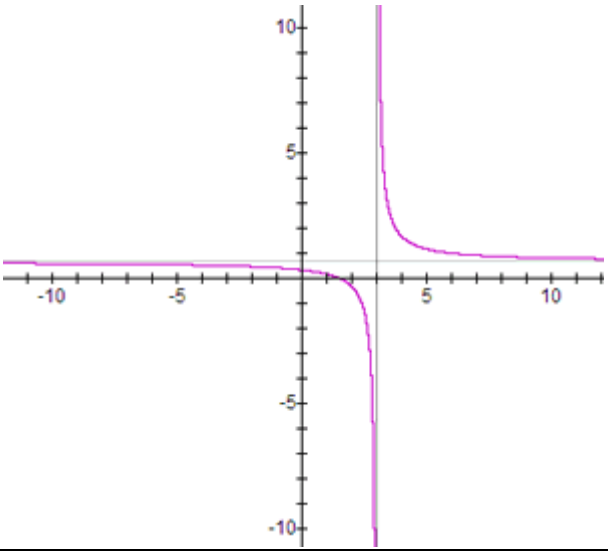
GRAPH B.



GRAPH C.



GRAPH D.



SHORT ANSWER, with work required to be shown, as indicated.

22. Let $f(x) = x + 2$ and $g(x) = \sqrt{8 - x}$.

(a) Find $\left(\frac{f}{g}\right)(-1)$. **Show work.**

(b) Find the domain of the quotient function $\frac{f}{g}$. **Explain.**

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23. Points (4, – 7) and (6, 3) are endpoints of the diameter of a circle.

(a) What is the length of the diameter? Give the exact answer, simplified as much as possible. **Show work.**
(b) What is the center point *C* of the circle?
(c) Given the point *C* you found in part (b), state the point symmetric to *C* about the *x*-axis.

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24. Find the equation for a line which passes through the points (2, 9) and (4, – 3). Write the equation in slope-intercept form. **Show work.**

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25. Alan, a resident of Metropolis, pays Metropolis an annual tax of $32 plus 2.6% of his annual income. If Alan paid $1,410 in tax, what was Alan's income? **Show work.**

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26. Let $f(x) = 2x^2 - 6$ and $g(x) = x - 5$.

(a) Find the composite function $(f \circ g)(x)$ and simplify. **Show work.**
(b) Find $(f \circ g)(-1)$. **Show work.**

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27. Find the exact solutions and simplify as much as possible: $3x^2 + 1 = 6x$. **Show work.**

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28. Given the function $f(x) = \frac{1}{3}x - 8$, find a formula for the inverse function. **Show work.**

29. The fuel efficiency F for a particular car is given by

$$F(x) = -0.016\,x^2 + 1.44\,x + 5.4$$

where x is the speed of the car in miles per hour (mph) and $F(x)$ is the corresponding fuel efficiency in miles per gallon (mpg).

(a) What is the fuel efficiency if the car's speed is 25 mph?

(b) What speed will yield the maximum fuel efficiency? **Show work.**

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30. Solve: $\frac{x-8}{x-5} + \frac{30}{x^2-25} = 0$. **Show work.**
