



UNIVERSITY OF  
LIVERPOOL

School of  
Management

## **Key Concept Overview**

Design of Experiments and  
Reliability

**KMGT 609**

**Total Quality Management**

**Week 6**

# Key Concept

## Design of Experiments and Reliability

### Design of experiments (DOE)

The principles of DOE are based on the observation that almost all types of operations or systems are affected by variables. It is important to understand the different levels of importance of these variables, investigate any interaction among them and identify the settings for these factors that will optimize performance of the process (Sower, 2011, pp.153-154).

From the concept of quality representing 'loss when products are produced that are not at the target specification' (Sower, 2011), Taguchi promoted an offline approach to the control of quality (as opposed to online approaches, such as sampling or SPC). The Taguchi techniques are aimed at designing products and processes that are immune to noise (variation).

This noise can be:

- Outer noise – environmental conditions
- Inner noise – deterioration of parts
- Between product noise – piece-to-piece variation due to manufacturing imperfections

For example, think about a process that manufactures shirts. The process would be subject to a number of variables, such as the type of threading used, quality of dyes, grade of cotton, speed of sewing etc. The decisions made on these variables will determine whether the shirts from that process will be able to resist environmental conditions (e.g. fading, shrinkage), deterioration of parts (e.g. loose seams) and piece-to-piece variations (e.g. some shirts are different from others).

Taguchi identified three main stages in the design process: system design, parameter design and tolerance design. The techniques associated with the design of experiments are primarily aimed at parameter design, where the main objective is to develop processes that are robust to noise variables.

Parameter design identifies two types of factors:

- Control factors
- Noise factors

The control factors are those process parameters that can be set and maintained, whereas noise factors cannot be directly controlled and yet still affect the quality

performance of the product. Parameter design aims to optimise the interaction between control and noise factors to produce robust processes.

Most products and processes comprise a number of factors, some or all of which may vary to give us quality problems. A product or process with (say) seven factors can only be optimised if we use the correct combination of factors even with only two levels of operation (Sower, 2011, pp.155-157).

For example, think about a process that bakes biscuits. The organisation needs to make decisions on a number of factors that will affect the quality of their product. These could include:

- Level of heat to bake at (low, medium or high)
- Length of time to bake for (long or short)
- Length of time to mix dough for (long or short)

In order to find the best settings, they would ideally need to try out different combinations of parameter settings that will give them the quality of product that they desire. One way of doing this would be to change one variable while other variables remain unchanged. This is typically known as *one-factor-at-a-time design*. However, this approach fails to take into account any interaction among the different factors. Another more complete and realistic approach is to use full factorial experimentation. However, even with two-level factors this approach can be extremely tedious (Sower, 2011, pp.158-163).

Taguchi's method proposes the use of *orthogonal arrays* to optimise the product or process design. This enables an organisation to identify the best parameter settings that make the product robust to noise while avoiding the cost and time associated with a full factorial experiment (Sower, 2011, pp.167-170). The arrays for seven factors that can each be set at two levels suggest the need to conduct eight experiments (as opposed to 128 experiments for a full factorial design). The array is shown below.

The control factors are considered through the evaluation of orthogonal arrays. Having undertaken the experiment, the average level of performance for each level of factor is then calculated. The optimum product or process settings are then selected. Individual factors that are not sensitive may then be optimised for cost.

(Logothetis and Wynn, 1989)

Number	A 1	B 2	C 3	D 4	E 5	F 6	G 7	Results
1	1	1	1	1	1	1	1	y <sub>1</sub>
2	1	1	1	2	2	2	2	y <sub>2</sub>
3	1	2	2	1	1	2	2	y <sub>3</sub>
4	1	2	2	2	2	1	1	y <sub>4</sub>
5	2	1	2	1	2	1	2	y <sub>5</sub>
6	2	1	2	2	1	2	1	y <sub>6</sub>
7	2	2	1	1	2	2	1	y <sub>7</sub>
8	2	2	1	2	1	1	2	y <sub>8</sub>

**L<sub>8</sub> (2<sup>7</sup>) Orthogonal Array**  
**7 Factors**  
**2 Levels each**  
**8 Experimental Runs**

### Reliability

Reliability is the probability that a product, piece of equipment or system will perform its intended function for a stated period of time under specified operating conditions (in effect, quality over time). The following should be taken into account when talking about reliability:

- Probability is expressed between 0 and 1
- Performance (functional and reliability failure)
- Time expectancy of the product
- Operating conditions (type and amount of usage/environment)

Reliability engineers distinguish between *inherent* (or *design*) and *achieved* reliability. In practice, reliability is determined by the number of failures per unit time (or failure rate). The reciprocal of the failure rate is often used in reliability computations. For non-repairable items,  $1/\lambda$  is defined as the mean time to failure (mean time between failures for repairable items). MTTF is the average time to failure for all items tested (Sower, 2011, pp.67-71). For items subject to mean time to failure, a failed item cannot be repaired and will be discarded (e.g. a broken lightbulb). For mean time between failures,

the failed item can be repaired (e.g. a mobile phone that has run out of charge), and this time represents the working time of the phone from full charge to empty charge.

By plotting a graph of the failure rate over time, the pattern of failure takes on the shape of a 'bathtub' curve characterised by three distinct phases (Sower, 2011, pp.67-71):

- Early life failure – due to design or raw material failures
- Constant failure (or random failure) – represents the normal working life of the product or system
- CD represents the wear-out failure – due to components or systems failing from wear after exceeding the design or intended life

### *Availability*

As a comprehensive measure of product dependability (i.e. its ability to function in the intended manner in spite of disturbances that may occur) the concept of **availability A(t)** is used.

Availability is a measure of the fraction of time a piece of equipment is expected to be operational (as opposed to being down for repairs). Availability ranges from 0 to 1.

Availability is a function of MTBF (mean time between failures) and MTTR (mean time to repair) and is determined by:

$$\frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

For example, the availability of desktop printers is increased by designing them in such a way that when they become unavailable (when the ink runs out), the cartridge can be replaced in a matter of minutes (i.e. the MTTR is low), after which the printer becomes available again.

### *Reliability of systems*

When a product or system consists of a number of independent components, the probability that that system or product will operate as planned requires the use of the rules of probability for independent events (Sower, 2011, pp.60-67). Systems can have three types of component configurations:

- in series
- in parallel
- in stand-by

*Series systems* – If two or more events are independent, and 'success' is defined as the probability that all of the events will occur, then the probability of success is equal to the product of the probabilities of the events.

Example: A room has two lamps, and both must work to give adequate lighting. If one lamp has a reliability of 0.9 and the other has a reliability of 0.8, the overall reliability is  $0.8 \times 0.9 = 0.72$ .

- $R(s) = R_1 \times R_2 \times \dots \times R_n$

Note that overall reliability is less than individual reliabilities.

*Parallel systems* – Products or systems with a large number of components can have overall reliability built in by applying *redundancy* in the design. This involves providing backup for some parts. If two events are independent and success is defined as the probability that at least one of the events will occur, the probability of success is equal to the probability of either one plus (1.00 minus that probability multiplied by the other probability). In the previous example, if only one lamp is required, the system reliability becomes  $0.90 + (1-0.90) \times 0.80 = 0.98$ . If a number of components are connected in parallel, then the procedure is evolved for each backup level. In the previous example, if a third lamp with a reliability of 0.70 is added and only one lamp is required for adequate lighting, the system reliability becomes:

- $0.90 + (1-0.90) \times 0.80 + (1-0.90) \times (1 - 0.80) \times 0.70 = 0.994$
- or  $R_s = 1 - (1-R_1)(1-R_2) \dots(1-R_n)$

*Standby systems* – In some applications, it is neither possible nor desirable to have components operating in parallel to improve reliability.

As an alternative, use is made of a standby system which is brought into operation when the original system fails. For example, consider the replacement of mains power generation with a standby diesel generator in case of mains power failure.

We consider the reliability of the system in time to be the probability of either component A surviving or A failing and B surviving. It is calculated in the same way as the reliability of parallel systems:

$$\text{Hence } R(s) = R_A + F_A \times R_B = R_A + (1-R_A) \times R_B$$

$R_A$  = Reliability of component A

$R_B$  = Reliability of component B

$F_A$  = (1-  $R_A$ ) or failure of component A

**References:**

Logothetis, N. & Wynn, H.P. (1989) *Quality through design: experimental design, off-line quality control, and Taguchi's contributions*. Oxford: Oxford University Press.

Sower, V. (2011), *Essentials of quality with cases and experiential exercises*. Hoboken, NJ: Wiley.