Problem Set 3

Due July 23 (in sections)

ARE/ECN 115A Summer Session 1, 2015

Last Name: _____ First Name: _____ Student ID:_____

Part A. Credit Markets

In part A of this problem set, we will think systematically about how the equilibrium interest rate is determined in the credit market. In particular, we will explore how the interest rate, the size of the economic surplus and the distribution of the surplus are affected by asymmetric information in the context of limited liability loans.

In the village of Quahog, people can choose to either farm or work in the factory. If a person works in the factory, she makes \$100 with certainty. People in Quahog are either born a SAFE farmer or a RISKY farmer. Half of the people are SAFE farmers and half are RISKY farmers. If either type of person chooses to farm, she will need \$300 investment in order to farm. Thus, the opportunity cost of farming is the \$100 one could have had earned if she instead worked in the factory. The only difference between SAFE and RISKY farmers is as follows:

- SAFE farmers have a good harvest all the time, they earn revenues of \$700 with 100% probability.
- RISKY farmers have a good harvest with 60% probability, in which they earn revenues of \$1100 with 60% probability; and they have a bad harvest with 40% probability, in which they earn revenues of \$0.

1 Perfect Information

Brian is a moneylender who lives in Quahog. His opportunity cost of money is 0.20 (i.e., he would earn 20% if he invested the money in a business instead of lending it to farmers). Brian offers limited liability loans, so a farmer does not have to repay the loan if she has a bad harvest. Since Brian lives in Quahog, he has perfect information about farmers. Specifically, he knows who is a SAFE farmer and who is a RISKY farmer.

1. Let Y_s be the farming income of a SAFE farmer and Y_R be the farming income of a RISKY farmer. The farming income for any famer Y is equal to revenues minus all costs (including opportunity cost). Derive expressions for the expected value of farming income $E(Y_S)$ and $E(Y_R)$ as functions of the interest rate i. Your expressions should take the form of $E(Y) = A + Bi$, where you have to find A and B.

   (a) What are the functions $E(Y_S)$ and $E(Y_R)$

   (b) Graph your functions $E(Y_S)$ and $E(Y_R)$; place i on the horizontal-axis; title the graph "Figure 1"

2. Let π be the profit of Brian the moneylender. Derive expressions for the expected value of profit for a loan to a safe farmer $E(\pi_S)$ and a loan to a risky farmer $E(\pi_R)$ as functions of the interest rate i. Your expressions should take the same form of $E(\pi) = A + Bi$, where you have to find A and B.

(a) What are the functions $E(\pi_S)$ and $E(\pi_R)$

(b) Graph your functions $E(\pi_S)$ and $E(\pi_R)$ in the same "Figure 1"

3. Using your questions and graph, answer the following questions:

   (a) What is the highest interest rate a SAFE farmer would be *willing to pay* for a loan from Brian?

   (b) What is the highest interest rate a RISKY farmer would be *willing to pay* for a loan from Brian?

   (c) What is the lowest interest rate Brian would be *willing to charge* on a loan to a SAFE farmer?

   (d) What is the lowest interest rate Brian would be *willing to charge* on a loan to a RISKY farmer?

4. First, assume that the loan market is *perfectly competitive*. There are many other lenders who would charge a lower interest than Brian, if Brian is making a profit.

   (a) What is the equilibirum interest rate Brian would charge a SAFE farmer?

   (b) What is the equilibrium interest rate Brian would charge a RISKY farmer?

   (c) What is Brian's total expected profit $E(\pi) = E(\pi_S) + E(\pi_R)$?

   (d) What is total expected income across all types of famers $E(Y) = E(Y_S) + E(Y_R)$?

5. Now, assume that all the other lenders left Quahog, and now Brian is a *monopolist* moneylender.

   (a) What is the equilibirum interest rate Brian would charge a SAFE farmer?

   (b) What is the equilibrium interest rate Brian would charge a RISKY farmer?

   (c) What is Brian's total expected profit $E(\pi) = E(\pi_S) + E(\pi_R)$?

   (d) What is total expected income across all types of famers $E(Y) = E(Y_S) + E(Y_R)$?

2 Asymmetric Information

Brian has decided to leave Quahog in search of a better life. Peter has come from a far away town, and has decided to live in Quahog. He is considering offering limited liability loans. Like Brian, Peter's opportunity cost of money is 0.20 (20%), and he is a *monopolist* since all the other moneylenders in Quahog have left. Peter, however, does not know the people in Quahog, so he cannot tell who was born a SAFE farmer and who was born a RISKY farmer. All he knows is that half of the people are SAFE and half are RISKY farmers. Thus, Peter can charge only one interest rate. In contrast to Brian, Peter suffers from asymmetric information. So when Peter thinks about the single interest rate he will charge, he must think about who will want the loan.

1. Let's think carefully about who will want a loan depending on the interest rate.

   (a) What is the maximum interest rate Peter can charge so that both types of farmers would want to borrow?

   (b) What is the maximum interest rate Peter can charge so that at least one type of farmer would want to borrow?

2. Now, let's think about Peter's expected profit function $E(\pi)$:

   (a) Derive an expression for $E(\pi)$ as a function of the interest rate i, for all interest rates below the i you identified in (2.1.a).

   (b) Derive an expression for $E(\pi)$ as a function of the interest rate i, for all interest rates above the i you identified in (2.1.a) and below the i you identified in (2.1.b)

   (c) Derive an expression for $E(\pi)$ as a function of the interest rate i, for all interest rates above the i you identified in (2.1.b).

3. Graph Peter's expected profit function $E(\pi)$ on a range of $i = 0$ to $i = 3$ (i.e. 0 to 300%)
4. Recall that Peter is a monopolist:
   (a) What is the equilibrium interest rate that Peter sets?
   (b) What is Peter's total expected profit $E(\pi) = E(\pi_S) + E(\pi_R)$?
   (c) What is total expected income across all types of famers $E(Y) = E(Y_S) + E(Y_R)$?

Part B. Risk and Informal Insurance

In part B of this problem set, we introduce risk preferences and study an alternative to formal insurance contracts. We evaluate how asymmetric information may also affect informal insurance arrangements.

3 Risk Preferences

There are three farmers in a village: Chris with utility function $U = C^{0.5} - 2$, Meg with utility function $U = C^2$, and Stewie with utility function $U = 15 + 2C$. All three farmers have certain wealth equal to \$100, and they earn random farm income Y which depends on the unknown level of pest infestation:

$$Y = \begin{cases} 200 & P = 2/5,\ Low\,Infestation \\ 75 & P = 2/5,\ Medium\,Infestation \\ 0 & P = 1/5,\ High\,Infestation \end{cases}$$

1. Find the following for Chris:
   (a) Expected utility
   (b) Certainty equivalent
   (c) Risk premium
   (d) Risk preference (risk loving, risk neutral or risk averse)?
2. Find the following for Meg:
   (a) Expected utility
   (b) Certainty equivalent
   (c) Risk premium
   (d) Risk preference (risk loving, risk neutral or risk averse)?
3. Find the following for Stewie:
   (a) Expected utility
   (b) Certainty equivalent
   (c) Risk premium
   (d) Risk preference (risk loving, risk neutral or risk averse)?
4. Quagmire is offering an insurance contract to farmers with premium equal to \$90. This insurance scheme pays out \$0 if there is low infestation, \$125 if there is medium infestation, and \$200 if there is high infestation. Quagmire knows perfectly well who Chris, Meg and Stewie are (i.e. there is no asymmetric information).
   (a) What is Quagmire's expected profit?
   (b) Will Chris want to purchase this insurance contract?
   (c) Will Meg want purchase this insurance contract?
   (d) Will Stewie want to purchase this insurance contract?

4 Informal Risk Sharing Arrangements

Quagmire, Meg and Stewie decide to leave the village. Chris stayed, and many other people migrated into the village. These new people are exactly like Chris: they have the same utility function $U = C^{0.5} - 2$, have the same certain wealth of \$100 and face the same random income Y (same amounts and probabilities).

We know that all these Chris-types would prefer some insurance, but no formal insurance is offered since Quagmire is gone. Thus, they discuss amongst each other and decided to implement an *informal risk sharing arrangement* (IRSA). All of the pest infestation risk in the village is idiosyncratic (that is, the risk is uncorrelated across all the people in the village).

Let T_L, T_M, T_H denote the transfer made by a farmer into the village insurance fund when that farmer has Low, Medium and High levels of pest infestation (a negative transfer means the farmer receives a payment). Assume that the transfers are out of income, not wealth. An optimal IRSA is a set of transfers T_L, T_M, T_H that satisfies the following two criteria: (1) **first-best**: it provides the maximum possible level of consumption smoothing (ideally it completely eliminates risk to consumption) and; (2) **affordable**: the expected value of transfers is zero for an individual (this means that, on average, the same amount of money is going into the village pot as is coming out of the village pot).

1. Find the values of T_L,T_M and T_H in an optimal IRSA.
2. Show that the values you specified in 4.1 are (a) first-best and (b) affordable (i.e. show that offering a higher value of consumption than your answer in 4.1 is not affordable).
3. What is the expected utility for Chris for this optimal IRSA?

5 Asymmetric Information

Now, assume that each Chris can now choose to relax by taking multiple breaks and not working hard on the farm. None of the people in the village can see whether a person is working hard or relaxing. If he relaxes, then three things change: his utility function, the random income amounts, and the probabilities. If he relaxes, his utility function becomes $U(C) = C^{0.5}$ since he does not incur the disutility of 2 units from working hard, and his random income is now given by the following:

$$Y = \begin{cases} 100 & P = 1/3,\ Low\,Infestation \\ 50 & P = 1/3,\ Medium\,Infestation \\ 0 & P = 1/3,\ High\,Infestation \end{cases}$$

1. First, assume that **no IRSA** is available.
   (a) What is the expected utility of relaxing on the farm?
   (b) Will Chris choose to work hard on the farm or relax on the farm?
2. Now, assume that the **IRSA you determined in (4.1)** was available.
   (a) What is the expected utility of relaxing on the farm?
   (b) Will Chris choose to work hard on the farm or relax on the farm?
3. Is the IRSA that you determined in (4.1) affordable if each Chris can choose to relax on the farm?