Problem Set #2
Due Monday March 20th, 2017
Spring 2017
Prof. Deza

PROBLEM SET #2

QUESTION 1: VALUE OF TIME, QUANTITY/QUALITY THEORY OF BECKER

Let's start with the standard model of fertility
N=number of children
X=consumption by parents
Y=primary earner income
I=total family income
t_c=time the secondary earner spends on each child
q=quality that must be given to each child
p_q=price of this quality
p_x=price of parental consumption goods
w=wage rate for secondary earner
U(x,n)= utility of the couple
T=total time per day (e.g. 24 hours)

Simplifying assumptions:

- Secondary earner does all the child care and primary earner does none
- Amount spent per child is fixed
- Amount of goods given per child is fixed
- Amount of work by the primary earner is fixed
- The secondary earner's allocation of time to child rearing or the labor market is the choice variable
- Children are divisible, so one can have 2.3 children

a) Write expressions for the secondary earner's income from work, the total family money income, the total family money expenditures and the family budget constraint.
b) What is the shadow price of a child (e.g. we saw price per child in class)? Write the expression and explain in words.
c) In a diagram, draw the budget constrain and indifference curve. Label the axes, the optimum (x*, n*) and n_{max}.
d) Suppose that the wages of the secondary earner decrease. In a separate diagram draw the new and old BC. Does the BC become flatter or steeper? Property label the gram.
e) Assume children are a normal good., and assume that income effect is larger in magnitude than the substitution effect. Decompose the shift from (x*,n*) to (x*',n*') into an income and substitution effect in a properly labeled graph. Label the intermediary optimum (x^m, n^m). Explain what drives the movements in the income and substitution effect. Does fertility increase or decrease in response to lower w?

f) Same as above, but now assume that the substitution effect is larger in magnitude than the income effect. Children are still are a normal good. Does fertility increase or decrease in response to this lower w
g) Now suppose that parents care about the quality of their children as well so that their utility function becomes U(x,n,q) where q is the quality of children. According to Becker's quantity and quality theory, what happens to the demand for n and q as the wages of the secondary earner increase? According to your graphs, is Becker's theory consistent with income effect being larger or smaller in magnitude than substitution effects?

QUESTION 2: PENSIONTHEORY

Assumptions are as follows

- Each person lives through two periods: working adult, retiree
- Nobody dies during the working period and everyone dies at the end of the retirement period.
- Population is growing at a rate such that the population of current worker is always 1.5 times the size of the population of current retirees.
- At time t, there are W_t workers and R_t retirees.
- Workers in period t become retirees in period t+1
- No saving, no storing
- Workers are able to produce 18 units of the good. Retirees produce only 3 units.
- Consumption of workers is C_w and consumption of retirees is C_r.

a) What is the ratio of workers to retirees?
b) What is the social budget constraint? Graph C_w on the x-axis and C_r on the y-axis. Draw the budget constraint and label it. Be sure to graph and label your graph clearly.
c) Find the point (C_w^e, C_r^e) that reflects the consumption mix if workers only consume what they produce and retirees only consume what they produce (The endowment point). Label it E in the budget line
d) Draw the indifference curve that corresponds to the endowment point, and the utility-maximizing indifference curve given this social budget constraint. Remenber that we assume that indifference curves are convex.
e) Now imagine that a policy intervention establishes PAYGO system with a defined retirement benefit of 6 units. How many units must each worker contribute to provide this benefit for each retiree?

QUESTION 3: PENSION THEORY (CONTINUATION of question 2)

Assumptions from Q2 apply to this problem.
Additionally, assume that

- The first period (age 25-50) is spent working and the second period (age 50-75) is spent on retirement.
- Assume that due to medical advances, life expectancy increases from 75 to 100 years.

- Remember that each generation is 1.5 times the size of the previous generation.
- The 75-100 year old retirees produce as much as the 50-75 year old retirees (3 units)

a) For the first 25 years after the medical innovation, the population is unstable as the first cohort of 75-year olds ages to 100. Afterwards it becomes stable again. What is the ratio of workers to retirees in the new stable population?
b) What is the new social budget constraint with these additional retirees? You do not need to draw it, just write the new social budget constraint.
c) To sustain the defined benefit 6 units per retiree, how many units must each worker contribute?

QUESTION 4:

Assumptions

- Each person lives through two periods: working adult, retiree
- Nobody dies during the working period and everyone dies at the end of the retirement period.
- Population is growing at a rate such that the population of current worker is always 2 times the size of the population of current retirees.
- At time t, there are W_t workers and R_t retirees.
- Workers in period t become retirees in period t+1
- No saving, no storing
- Workers cultivate the land and their per capital production is equivalent to 24 units of the good. The retirees cannot cultivate the land and they can only gather from nature, which is equivalent to 8 per capita units of the good.
- Consumption of workers is C_w and consumption of retirees is C_r.

a) Derive the budget constraint , write the equation and draw it on the graph. Find the endowment point $(C^e_{w,}, C^e_r)$ that reflects the consumption mix if workers only consume what they produce and retirees only consume what they produce. Label this point "E" for endowment. Draw the indifference curve that goes through the endowment point and an optimum indifference curve.
b) Consider now a population with the same growth rate as the one under baseline assumptions (each generation is two times bigger than the previous one), but only half of the workers survive from working age to retirement age.
   -How many workers will be there for every retiree?
   -Derive and draw the new social budget constraint for this situation

QUESTION 5:

Assumptions (same as question 4)

- Each person lives through two periods: working adult, retiree
- Nobody dies during the working period and everyone dies at the end of the retirement period.

- Population is growing at a rate such that the population of current worker is always 2 times the size of the population of current retirees.
- At time t, there are W_t workers and R_t retirees.
- Workers in period t become retirees in period t+1
- No saving, no storing
- Workers cultivate the land and their per capital production is equivalent to 24 units of the good. The retirees cannot cultivate the land and they can only gather from nature, which is equivalent to 8 per capita units of the good.
- Consumption of workers is C_w and consumption of retirees is C_r.

a) Assume that fertility falls to a level such that the population merely reproduces itself ($W_t = R_t$) and from then on it has a growth rate of zero. Derive the new social budget constraint for this situation and draw it in a diagram.
b) Now imagine that a policy intervention establishes a PAYGO system with defined benefits of 4 units. Please compute how much each current worker would have to contribute (compared to the situation in which workers only consume what they produce and retirees only consume what they produce) to provide this benefit for each current retiree in the following three situations. You need to compute three values
   -working population is twice as big as retired population
   -working population is twice as big as retired population, but only half of the workers survive from working age to retirement age (they live throughout the working age).
   -population merely replaces itself ($W_t = R_t$)

QUESTION 6:ECONOMICS OF AGING AND RETIREMENT

Gruber and Wise claim that the decline of labor force participation among the older people is "perhaps the most dramatic feature of labor-force change over the past several decades."

a) Suppose the median full-time worker is considering whether to retire next year when he or she turns 62 or continue working one more year. The worker's annual salary last year was 45,113.Suppose this worker is self-employed. For workers who are self-employed, payroll taxes for social security are 12.4% applied to earnings up to $94,200. In terms of payroll taxes alone, if social security benefits are defined to be 90% of wages each year, should this person retire nest year? Why or why not? (Ignore federal and state taxes). Calculate the implicit tax
b) If the individual is not self-employed, but instead works for a large Fortune 500 company, the payroll taxes are 6.2% of earnings up to 94,200. Based on the payroll taxes alone, should this person retire next year? Why or why not? Calculate the implicit tax.